SUBPROJECT MTM2014-59179-C2-1-P

Title

Fundamentals, methods and applications of continuous optimization

Summary of the coordinated Project

Optimization is an important subject in mathematics since ancient times. Solving continuous optimization problems was the main motivation for the development of calculus in the XVII Century. Since then, continuous optimization allowed scientists to model in natural sciences during three centuries and, after the irruption of the computers in the mid 1900s, became the black box in decision making, with very important applications in operations research (also called management science), economics, engineering, statistics, etc. Classical calculus deals with differentiable real-valued functions of finitely many variables, but in many real applications the objective function is either vector-valued or set-valued and, when it is real valued, it is non-smooth and possibly non-convex. Moreover, the objective function (as well as the constraints) may suffer perturbations due to error observations or the inherent uncertainty of the data. For this reason, new branches of mathematics as stability analysis, set-valued analysis, extended derivatives, monotone operators, etc. (altogether known as variational analysis after the title of a famous book by R.T. Rockafellar and R.J.-B. Wets, whose 1st edition was published in 1998) were developed during the last decades.

This project has five main objectives:

1. Analyze the fundamentals of optimization as convexity, subdifferential calculus, monotonicity, and self-contracted curves.

2. To get information on the feasible and the optimal sets from the data.

3. Characterize the global minima of optimization problems and provide accurate lower bounds for the optimal value.

4. Quantify the stability of optimization problems under data perturbations.

5. Provide efficient numerical methods for complex feasibility and optimization problems and their application to real-world problems.

Summary of objectives

Remark: the objectives of the UA (UMH) coordinated team are marked in blue (red, respectively).

1. Fundamentals of optimization.

1.1 Decomposition of non-closed convex sets.

1.2 New results on evenly convex sets and functions.

1.3 Paramonotone operators.

1.4 Representations of quasimonotone operators.

1.5 Outer limits of subdifferentials.

1.6 Subdifferentials and coderivatives of multifunctions.

2. Inequalities.

2.1 Existence theorems for lexicographical linear systems.

2.2 Characterization of the solution sets of lexicographical linear systems.

2.3 Systems involving vector-valued/set-valued mappings functions.

2.4 Complementarity problems and other models beyond convexity.

3. Optimality and duality.

3.1 Duality in convex optimization and applications.

3.2 Duality in evenly convex optimization.

3.3 Optimality in vector semi-infinite optimization.

3.4 Optimality in robust optimization.

4. Stability.

4.1 Calmness and Lipschitz moduli under perturbations of all data.

4.2 Calmness modulus and critical objective size.

4.3 Moduli under structured perturbations.

4.4 Calmness under non-uniqueness of optimal solutions.

4.5 Lower Lipschitz stability.

4.6 Variational behavior of the optimal value function.

4.7 Hölder stability.

4.8 Stability analysis via monotonicity.

5. Optimization methods.

5.1 Relaxation methods for semi-infinite systems.

5.2 Projection algorithms for non-convex systems.

5.3 A new class of proximal point methods.

5.4 New algorithms for global optimization.

5.5 Generalized sweeping processes.

5.6 New algorithms for optimization problems arising in biochemistry.

5.7 Optimization methods for problems arising in engineering.