Towards a common framework for Mathematics degrees in Europe

This paper reflects the unanimous consensus of the mathematics group of the project "Tuning educational structures in Europe", but it has not yet been discussed with the wider community of European mathematicians. Since the group does not pretend to have any representative role, we insist that any kind of action along the lines we sketch will require a much broader agreement.

Summary

- This paper refers only to universities (including technical universities), and none of our proposals apply to other type of institutions.
- The aim of a "common framework for Mathematics degrees in Europe" is to facilitate an automatic recognition of degrees in order to help mobility.
- The idea of a common framework must be combined with an accreditation system.
- The two components of a common framework are similar (although not necessarily identical) structures and a basic common core curriculum (allowing for some degree of local flexibility) for the first two or three years.
- Beyond the basic common core curriculum, and certainly in the second cycle, programmes could diverge significantly. Since there are many areas in Mathematics, and many of them are linked to other fields of knowledge, flexibility is of the utmost importance.
- Common ground for all programmes will include calculus in one and several real variables and linear algebra.
- We propose a broad list of further areas that graduates should be acquainted with in order to be easily recognised as mathematicians. It is not necessary that all programmes include individual modules covering each of these areas.
- We do not present a prescriptive list of topics to be covered, but we do mention the three skills that any mathematics graduate should acquire:
  (a) the ability to conceive a proof,
  (b) the ability to model a situation,
  (c) the ability to solve problems.
- The first cycle should allow time to learn some computing and to meet at least one major area of application of Mathematics.
- We should aim for a wide variety of flavours in second cycle programmes in mathematics. Their unifying characteristic feature should be the requirement that all students carry on a significant amount of individual work. To do this, a minimum of 300 ECTS credits should be necessary to obtain a Bachelor + Master.
- It might be acceptable that various non-identical cycle structures coexist, but large deviations from the standard should be grounded in appropriate entry level requirements, or other program specific factors, which can be judged by external accreditation. Otherwise, such degrees risk not benefiting from the automatic European recognition provided by a common framework, even though they may constitute worthy higher education degrees.

1. A common framework: what it should and shouldn't be or do

1.1 The only possible aim to agree on a "common European framework" should be to facilitate an automatic recognition of mathematics degrees in Europe in order to help mobility. By this we mean that when somebody with a degree in mathematics from country A goes to country B:

a) He/she will be legally recognised as holding such a degree, and the Government of country B will not require further proof of competence.

b) A potential employer in country B will be able to assume that he/she has the general knowledge expected from somebody with a mathematics degree.

Of course, neither of these guarantees employment: the mathematics graduate will still have to go through whatever procedures (competitive exams, interviews, analysis of his/her curriculum, value of the degree awarding institution in the eyes of the employer,...) are used in country B to get either private or public employment.

1.2 One important component of a common framework for the mathematics degrees in Europe is that all programs have similar (although not necessarily identical) structures. Another component is agreeing on a basic common core curriculum (allowing for some degree of local flexibility).

1.3 We should emphasise that by no means do we think that agreeing on any kind of common framework can be used as a tool for automatic transfer between Universities. These will always require consideration by case, since different programmes can get students to adequate levels in different but coherent ways, but an inappropriate mixing of programmes may not.

1.4 In many European countries there exist higher education institutions that differ from universities both on the level they demand from students and in their general approach to teaching and learning. In fact, to avoid excluding from higher education a substantial number of students, it is essential to keep the differences. We want to make explicit that **this paper refers only to universities (including technical universities)**, and that any proposal of a common framework designed for universities would not apply to other type of institutions.

2. Towards a common core mathematics curriculum

2.1 General remarks

At first sight, mathematics seems to be well suited for the definitions of a core curriculum, e.g. for the first two or three years. Because of the very nature of mathematics, and its logical structure, there will be a common part in all mathematics programmes, consisting of the fundamental notions. On the other hand, there are many areas in Mathematics, and many of them are linked to other fields of knowledge (computer science, physics, engineering, economics, etc.). Flexibility is of the utmost importance to keep this variety and the interrelations that enrich our science.

There could possibly be an agreement on a list of subjects that must absolutely be included (linear algebra, calculus/analysis) or that should be included (probability/statistics, some familiarity with the mathematical use of a computer) in any mathematics degree. In the case of some specialised courses, such as mathematical physics, there will certainly be variations between countries and even between universities within one country, without implying any difference of quality of the programmes.

Moreover, a large variety of mathematics programmes exist currently in Europe. Their entry requirements vary, as do their length and the demands on the student. It is extremely important that this variety be maintained, both for the efficiency of the education system and socially, to accommodate the possibilities of more potential students. To fix a single definition of contents, skills and level for the whole of European higher education would exclude many students from the system, and would, in general, be counterproductive.

In fact, the group is in complete agreement that programmes could diverge significantly beyond the basic common core curriculum (e.g. in the direction of "pure" mathematics, or probability - statistics applied to economy or finance, or mathematical physics, or the teaching of mathematics in secondary schools). The presentation and level of rigour, as well as accepting there is and must continue to be variation in emphasis and, to some extent, content, even within the first two or three years, will make all those programmes recognisable as valid mathematics programmes.

As for the second cycle, not only do we think that different programs could differ, but we are convinced that, to reflect the diversity of mathematics and its relations with other fields, all kinds of different second cycles in mathematics should be developed at different institutions.

2.2 The need for accreditation

The idea of a basic core curriculum must be combined with an accreditation system. If the aim is to recognise that various universities fulfil the requirement of the core curriculum, then one has to check on three aspects:

- a list of contents
- a list of skills
- the level of mastery of concepts

These cannot be reduced to a simple scale.

To give accreditation to a mathematics programme, an examination by a group of peer reviewers, mostly mathematicians, is necessary. The key aspects to be evaluated should be:

(a) the programme as a whole
(b) the units in the programme (both the contents and the level)
(c) the entry requirements
(d) the learning outcomes (skills and level attained)
(e) a qualitative assessment by both graduates and employers

The group does not believe that a (heavy) system of European accreditation is needed, but that universities in their quest for recognition will act at the national level. For this recognition to get international value, mathematicians from other countries must be included among the reviewers.

3. A common core curriculum and the Bologna agreement

How various countries implement the Bologna agreement will make a difference on core curricula. In particular, 3+2 may not be equivalent to 5, because, in a 3+2 years structure, the 3 years could lead to a professional diploma, meaning that less time is spent or fundamental notions, or to a supplementary 2 years, and in that case the whole spirit of the 3 years programme should be different.

The group did not attempt to solve contradictions that could appear in the case of different implementations of the agreement (i.e. if three years and five years university programmes coexist; or different cycle structures are established: 3+1, 3+2, 4+1, 4+2 have all been proposed). As we said before, it might be acceptable that various systems coexist, but we believe that large deviations from the standard (such as a 3+1 structure) should be grounded in appropriate entry level requirements, or other program specific factors, which can be judged by external accreditation. Otherwise, such degrees risk not benefiting from the automatic European recognition provided by a common framework, even though they may constitute worthy higher education degrees.

4. Some principles for a common core curriculum for the first degree (Bachelor) in mathematics

We do not feel that fixing a detailed list of topics to be covered is necessary, or even convenient. But we think that it is possible to give some guidelines as to the common contents of a "European first degree in Mathematics", and more important, as to the skills that all graduates should develop.

4.1 Contents

4.1.1 All mathematics graduates will have knowledge and understanding of, and the ability to use, mathematical methods and techniques appropriate to their programme. Common ground for all programmes will include

- calculus in one and several real variables
- linear algebra.

4.1.2 Mathematics graduates must have knowledge of the basic areas of mathematics, not only those that have historically driven mathematical activity, but also others of more modern origin. Therefore graduates should be acquainted with most, preferably all, of the following:

- basic differential equations
- basic complex functions
- some probability
- some statistics
- some numerical methods
- basic geometry of curves and surfaces
- some algebraic structures
- some discrete mathematics

These need not be learned in individual modules covering each subject in depth from an abstract point of view. For example, one could learn about groups in a course on (abstract) group theory or in the framework of a course on cryptography. Geometric ideas, given their central role, could appear in a variety of courses.

4.1.3 Other methods and techniques will be developed according to the requirements and character of the programme, which will also largely determine the levels to which the developments are taken. In any case, all programmes should include a substantial number of courses with mathematical content.

4.1.4 In fact, broadly two kinds of mathematics curricula currently coexist in Europe, and both are useful. Let us call them, following [QAA][1]:, “theory-based” and “practice-based” programmes. The weight of each of the two kinds of programmes varies widely depending on the country, and it might be interesting to find whether most European university programmes of mathematics are "theory-based" or not.

Graduates from theory-based programmes will have knowledge and understanding of results from a range of major areas of mathematics. Examples of possible areas are algebra, analysis, geometry, number theory, differential equations, mechanics, probability theory and statistics, but there are many others. This knowledge and understanding will support the knowledge and understanding of mathematical methods and techniques, by providing a firmly developed mathematical context.

Graduates from practice-based programmes will also have knowledge of results from a range of areas of mathematics, but the knowledge will commonly be designed to support the understanding of models and how and when they can be applied. Besides those mentioned above, these areas include numerical analysis, control theory, operations research, discrete mathematics, game theory and many more. (These areas may of course also be studied in theory-based programmes.)

4.1.5 It is necessary that all graduates will have met at least one major area of application of their subject in which it is used in a serious manner and this is considered essential for a proper appreciation of the subject. The nature of the application area and the manner in which it is studied might vary depending on whether the programme is theory-based or practice-based. Possible areas of application include physics, astronomy, chemistry, biology, engineering, computer science, information and communication technology, economics, accountancy, actuarial science, finance and many others.

4.2 Skills

4.2.1 For a standard notion like integration in one variable, the same “content” could imply:

- computing simple integrals
- understanding the definitions of the Riemann integral
- proving the existence and properties of the Riemann integral for classes of functions
- using integrals to model and solve problems of various sciences.

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[1] [QAA]The *Benchmark document on Mathematics, Statistics and Operational Research*, from the UK Quality Assurance Agency for Higher Education (http://www.qaa.ac.uk/crntwork/benchmark/phase2/mathematics.pdf), was considered extremely useful and met with unanimous agreement from the group. In fact we have quoted it almost verbatim at some points.

So on one hand the contents must be clearly spelled out, and on the other various skills are developed by the study of the subject.

4.2.2 Students who graduate from programmes in mathematics have an extremely wide choice of career available to them. Employers greatly value the intellectual ability and rigour and the skills in reasoning that these students will have acquired, their firmly established numeracy, and the analytic approach to problem-solving that is their hallmark.

Therefore, the three key skills that any mathematics graduate should acquire are:

(a) the ability to conceive a proof,
(b) the ability to model a situation,
(c) the ability to solve problems.

It is clear that, nowadays, solving problems should include their numerical and computational resolution. This requires a sound knowledge of algorithms and programming and the use of the existing software.

4.2.3 Note also that skills and level are developed progressively through the practice of many subjects. We do not start a mathematics programme with one course called "how to make a proof" and one called "how to model a situation", with the idea that those skills will be acquired immediately. Instead, it is through practice in all courses that these develop.

4.3 Level

All graduates will have knowledge and understanding developed to higher levels in particular areas. The higher-level content of programmes will reflect the title of the programme. For example, graduates from programmes with titles involving statistics will have substantial knowledge and understanding of the essential theory of statistical inference and of many applications of statistics. Programmes with titles such as mathematics might range quite widely over several branches of the subject, but nevertheless graduates from such programmes will have treated some topics in depth.

5. The second degree (Masters) in mathematics

We have already made explicit our belief that establishing any kind of common curriculum for second cycle studies would be a mistake. Because of the diversity of mathematics, the different programmes should be directed to a broad range of students, including in many cases those whose first degree is not in mathematics, but in more or less related fields (computer science, physics, engineering, economics, etc.). We should therefore aim for a wide variety of flavours in second cycle programmes.

Rather than the contents, we think that the common denominator of all second cycles should be the level of achievement expected from students. A unifying characteristic feature could be the requirement that all second cycle students carry on a significant amount of individual work. This could be reflected in the presentation of a substantial individual project.

We believe that, to achieve the level necessary to do real individual work in mathematics, the time required to obtain both degrees (Bachelor + Master) should be the equivalent of at least

300 ECTS credits. Arguments justifying exceptions to this minimum should be supported by external accreditation.

Whether this 300 ECTS credits should be split in a 3 years Bachelor, followed by a 2 years Masters, or whether a 4+1 structure is preferable, may depend on a number of circumstances. For example, a 3+2 break up will surely facilitate crossing between fields, where students pursue Masters in areas different from that were they got their Bachelors.

One aspect that can not be ignored, at least in mathematics, is the training of secondary school teachers. If the pedagogical qualification must be obtained during the first cycle studies, they should probably last for 4 years. On the other hand, if secondary school teaching requires a Masters (or some other kind of postgraduate qualification), a 3+2 structure may be adequate.