Departamento de Matemáticas

A closure operator for clopen topologies

Abstract: A topology τ on a nonempty set X is called a *clopen topology* provided each member of τ is both open and closed. Such topologies have been called both locally indiscrete and indiscretely generated in the literature. In joint work with Colin Bloomfield, to appear, *Bull. Belgian Math. Soc.*, we show how such topologies arise from a natural closure operator familiar to any student of mathematics.

Given a function f from X to Y, the operator $E \mapsto f^{-1}(f(E))$ is a closure operator on the power set of X whose fixed points are the closed subsets corresponding to a clopen topology on X. Conversely, for each clopen topology τ on X, we produce a function f with domain X such that $\tau = \{E \subseteq X : E = f^{-1}(f(E))\}$.

We characterize the clopen topologies on X as those that are weak topologies determined by a surjective function with values in some discrete topological space. Paralleling this result, we show that a topology admits a clopen base if and only if it is a weak topology determined by a family of functions with values in discrete spaces, gaining a different perspective on an embedding theorem of Vedesinoff.

Finally, we consider the operator $E \mapsto f^{-1}(f(E))$ as a potential interior operator on the power set of Y.

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